Signature

Notations

Traditional name

Levi–Civita symbol

Traditional notation

$¶_{n_1, n_2, ¼ , n_d}$

Mathematica StandardForm notation

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Signature@8n1, n2, ¼, nd<D
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Primary definition

04.21.02.0001.01

$¶_{n_1, n_2, ¼ , n_d}$ Š H- 1L^t

where t is the number of permutations from the sorted version of 8n_1, n_2, ¼ , n_d< to 8n_1, n_2, ¼ , n_d<.

04.21.02.0002.01

$¶_{n_1, n_2, ¼ , n_d}$ Š 0 •; n_i Š n_j ì 1 £ i £ d ì 1 £ j £ d

Specific values

Values at fixed points

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$¶_0$ Š 1

04.21.03.0002.01

$¶_n$ Š 1

04.21.03.0003.01

$¶_{1, 1}$ Š 0

04.21.03.0004.01

$¶_{1, 2}$ Š 1

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$¶_{2, 1}$ Š - 1

04.21.03.0006.01

$¶_{z, a}$ Š - 1

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$\epsilon_{a,z} = 1$

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$\epsilon_{1,1,2} = 0$

04.21.03.0009.01

$\epsilon_{1,2,3} = 1$

04.21.03.0010.01

$\epsilon_{1,3,2} = -1$

04.21.03.0011.01

$\epsilon_{2,3,1} = 1$

04.21.03.0012.01

$\epsilon_{2,1,3} = -1$

04.21.03.0013.01

$\epsilon_{3,1,2} = 1$

04.21.03.0014.01

$\epsilon_{3,2,1} = -1$

04.21.03.0015.01

$\epsilon_{1,2,3,4} = 1$

04.21.03.0016.01

$\epsilon_{1,2,4,3} = -1$

General characteristics

Domain and analyticity

$\epsilon_{n_1, n_2, \ldots, n_d}$ is a nonanalytical function, defined on the set of tuples of integers with possible values $0, \pm 1$.

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$(n_1 * n_2 * \ldots * n_d) \in \epsilon_{n_1, n_2, \ldots, n_d} \wedge \mathbb{Z}^n \in \{-1, 0, 1\}$

Symmetries and periodicities

Quasi-permutation symmetry

04.21.04.0002.01

$\epsilon_{n_1, n_2, \ldots, n_d} = -\epsilon_{n_2, n_1, \ldots, n_d}$

04.21.04.0003.01

$\epsilon_{n_1, n_2, \ldots, n_k, \ldots, n_j, \ldots, n_d} = -\epsilon_{n_1, n_2, \ldots, n_j, \ldots, n_k, \ldots, n_d}$

04.21.04.0004.01

$\epsilon_{n_1, n_2, \ldots, n_d} = (-1)^{d+1} \epsilon_{n_2, n_3, \ldots, n_d, n_1}$

Transformations

Products, sums, and powers of the direct function

Products of the direct function

04.21.16.0001.01

$$\epsilon_{n_1,n_2,\ldots,n_d}\,\epsilon_{m_1,m_2,\ldots,m_d} = -\sum_{\text{permutations}(m_1,m_2,\ldots,m_d)} \epsilon_{m_1,m_2,\ldots,m_d} \prod_{k=1}^{d} \delta_{n_k,m_k}$$

04.21.16.0002.01

$$\epsilon_{n_1,n_2,\ldots,n_{r-1},n_r,n_{r+1},\ldots,n_d}\,\epsilon_{n_1,n_2,\ldots,n_{r-1},m_r,m_{r+1},\ldots,m_d} = -\frac{(d-r)!\,r!}{d!} \sum_{\text{permutations}(m_r,m_{r+1},\ldots,m_d)} \epsilon_{m_r,m_{r+1},\ldots,m_d} \prod_{k=r}^{d} \delta_{n_k,m_k}$$

Complex characteristics

Real part

04.21.19.0001.01

$$\operatorname{Re}(\epsilon_{n_1,n_2,\ldots,n_d}) = \epsilon_{n_1,n_2,\ldots,n_d}$$

Imaginary part

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$$\operatorname{Im}(\epsilon_{n_1,n_2,\ldots,n_d}) = 0$$

Absolute value

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$$|\epsilon_{n_1,n_2,\ldots,n_d}| = \sqrt{\epsilon_{n_1,n_2,\ldots,n_d}^{\,2}}$$

Argument

04.21.19.0004.01

$$\arg(\epsilon_{n_1,n_2,\ldots,n_d}) = \tan^{-1}(\epsilon_{n_1,n_2,\ldots,n_d},\,0)$$

Conjugate value

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$$\overline{\epsilon_{n_1,n_2,\ldots,n_d}} = \epsilon_{n_1,n_2,\ldots,n_d}$$

Summation

Finite summation

04.21.23.0001.01

$$\sum_{t_1=1}^{n}\sum_{t_2=1}^{n}\ldots\sum_{t_r=1}^{n} \epsilon_{t_1,t_2,\ldots,t_r,n_{r+1},\ldots,n_n}\,\epsilon_{t_1,t_2,\ldots,t_r,m_{r+1},\ldots,m_n} = r!\sum_{m_{r+1}=1}^{n}\sum_{m_{r+2}=1}^{n}\ldots\sum_{m_n=1}^{n} \epsilon_{m_{r+1},\ldots,m_n} \prod_{k=r+1}^{n} \delta_{n_k,m_k}$$

Theorems

The determinant of a matrix with entries elements

The determinant $\det(A)$ of the $n \times n$ matrix A with entries $a_{i\,j}$ can be expressed as

$$\det(A) = \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \ldots \sum_{k_n=1}^{n} \epsilon_{k_1\,k_2,\ldots,k_n}\, a_{1\,k_1}\, a_{2\,k_2} \ldots a_{n\,k_n}.$$

Antisymmetric Levi-Civita tensor

The set of all $\{\epsilon_{n_1,n_2,\ldots,n_d} ;\ 1 \leq n_1 \leq d,\ \ldots,\ 1 \leq n_d \leq d\}$ forms the completly antisymmetric Levi-Civita tensor of dimension d.

History

– T. Levi–Civita (1896)

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