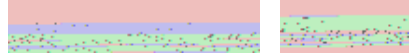


Signature



Notations

Traditional name

Levi-Civita symbol

Traditional notation

$\epsilon_{n_1, n_2, \dots, n_d}$

Mathematica StandardForm notation

Signature@8n₁, n₂, ..., n_d<D

Primary definition

04.21.02.0001.01

$\epsilon_{n_1, n_2, \dots, n_d} = \text{sgn}(\pi)$

where t is the number of permutations from the sorted version of n_1, n_2, \dots, n_d to n_1, n_2, \dots, n_d .

04.21.02.0002.01

$\epsilon_{n_1, n_2, \dots, n_d} = \prod_{i < j} \text{sgn}(n_j - n_i)$

Specific values

Values at fixed points

04.21.03.0001.01

$\epsilon_0 = 1$

04.21.03.0002.01

$\epsilon_n = 1$

04.21.03.0003.01

$\epsilon_{1,1} = 0$

04.21.03.0004.01

$\epsilon_{1,2} = 1$

04.21.03.0005.01

$\epsilon_{2,1} = -1$

04.21.03.0006.01

$\epsilon_{2,a} = -1$

04.21.03.0007.01
 $\mathfrak{F}_{d,z} \check{S} 1$

04.21.03.0008.01
 $\mathfrak{F}_{1,1,2} \check{S} 0$

04.21.03.0009.01
 $\mathfrak{F}_{1,2,3} \check{S} 1$

04.21.03.0010.01
 $\mathfrak{F}_{1,3,2} \check{S} - 1$

04.21.03.0011.01
 $\mathfrak{F}_{2,3,1} \check{S} 1$

04.21.03.0012.01
 $\mathfrak{F}_{2,1,3} \check{S} - 1$

04.21.03.0013.01
 $\mathfrak{F}_{3,1,2} \check{S} 1$

04.21.03.0014.01
 $\mathfrak{F}_{3,2,1} \check{S} - 1$

04.21.03.0015.01
 $\mathfrak{F}_{1,2,3,4} \check{S} 1$

04.21.03.0016.01
 $\mathfrak{F}_{1,2,4,3} \check{S} - 1$

General characteristics

Domain and analyticity

$\mathfrak{F}_{n_1, n_2, \frac{1}{4}, n_d}$ is a nonanalytical function, defined on the set of tuples of integers with possible values 0, ±1.

04.21.04.0001.01
 $\text{Re} \mathfrak{F}_{n_1, n_2, \frac{1}{4}, n_d} \in \mathbb{Z}^n \text{ \& } \{ -1, 0, 1 \}$

Symmetries and periodicities

Quasi-permutation symmetry

04.21.04.0002.01
 $\mathfrak{F}_{n_1, n_2, \frac{1}{4}, n_d} \check{S} - \mathfrak{F}_{n_2, n_1, \frac{1}{4}, n_d}$

04.21.04.0003.01
 $\mathfrak{F}_{n_1, n_2, \frac{1}{4}, n_k, \frac{1}{4}, n_j, \frac{1}{4}, n_d} \check{S} - \mathfrak{F}_{n_1, n_2, \frac{1}{4}, n_j, \frac{1}{4}, n_k, \frac{1}{4}, n_d}$

04.21.04.0004.01
 $\mathfrak{F}_{n_1, n_2, \frac{1}{4}, n_d} \check{S} \text{ H } 1 \text{ L}^{d+1} \mathfrak{F}_{n_2, n_3, \frac{1}{4}, n_d, n_1}$

Transformations

Products, sums, and powers of the direct function

Products of the direct function

04.21.16.0001.01

$$\Gamma_{n_1, n_2, \frac{1}{4}, n_d} \Gamma_{m_1, m_2, \frac{1}{4}, m_d} \tilde{S} - \hat{a} \prod_{\text{permutations } M} \Gamma_{m_1, m_2, \frac{1}{4}, m_d} \prod_{k=1}^d d_{n_k, m_k}$$

04.21.16.0002.01

$$\Gamma_{n_1, n_2, \frac{1}{4}, n_{r-1}, n_r, n_{r+1}, \frac{1}{4}, n_d} \Gamma_{n_1, n_2, \frac{1}{4}, n_{r-1}, m_r, m_{r+1}, \frac{1}{4}, m_d} \tilde{S} - \frac{H! - r! r!}{d!} \hat{a} \prod_{\text{permutations } M} \Gamma_{m_r, m_{r+1}, \frac{1}{4}, m_d} \prod_{k=r}^d d_{n_k, m_k}$$

Complex characteristics

Real part

04.21.19.0001.01

$$\text{Re} \Gamma_{n_1, n_2, \frac{1}{4}, n_d} \tilde{S} \Gamma_{n_1, n_2, \frac{1}{4}, n_d}$$

Imaginary part

04.21.19.0002.01

$$\text{Im} \Gamma_{n_1, n_2, \frac{1}{4}, n_d} \tilde{S} = 0$$

Absolute value

04.21.19.0003.01

$$|\Gamma_{n_1, n_2, \frac{1}{4}, n_d} \tilde{S}| = \sqrt{|\Gamma_{n_1, n_2, \frac{1}{4}, n_d}|^2}$$

Argument

04.21.19.0004.01

$$\arg \Gamma_{n_1, n_2, \frac{1}{4}, n_d} \tilde{S} = \tan^{-1} \Gamma_{n_1, n_2, \frac{1}{4}, n_d}, 0M$$

Conjugate value

04.21.19.0005.01

$$\overline{\Gamma_{n_1, n_2, \frac{1}{4}, n_d} \tilde{S}} = \Gamma_{n_1, n_2, \frac{1}{4}, n_d}$$

Summation

Finite summation

04.21.23.0001.01

$$\hat{a} \hat{a} \frac{1}{4} \hat{a} \prod_{t_1=1}^n \prod_{t_2=1}^n \prod_{t_r=1}^n \Gamma_{t_1, t_2, \frac{1}{4}, t_r, m_{r+1}, \frac{1}{4}, m_n} \Gamma_{t_1, t_2, \frac{1}{4}, t_r, m_{r+1}, \frac{1}{4}, m_n} \tilde{S} r! \hat{a} \hat{a} \frac{1}{4} \hat{a} \prod_{m_{r+1}=1}^n \prod_{m_{r+2}=1}^n \prod_{m_r=1}^n \Gamma_{m_{r+1}, \frac{1}{4}, m_n} \prod_{k=r+1}^n d_{n_k, m_k}$$

Theorems

The determinant of a matrix with entries elements

The determinant $\det A$ of the $n \times n$ matrix A with entries a_{ij} can be expressed as

$$\det A = \sum_{k_1=1}^n \sum_{k_2=1}^n \dots \sum_{k_n=1}^n \epsilon_{k_1 k_2 \dots k_n} a_{1 k_1} a_{2 k_2} \dots a_{n k_n}.$$

Antisymmetric Levi-Civita tensor

The set of all $\epsilon_{k_1, k_2, \dots, k_d}$, $1 \leq k_1 \leq d, 1 \leq k_2 \leq d, \dots, 1 \leq k_d \leq d$ forms the completely antisymmetric Levi-Civita tensor of dimension d .

History

– T. Levi-Civita (1896)

Copyright

This document was downloaded from functions.wolfram.com, a comprehensive online compendium of formulas involving the special functions of mathematics. For a key to the notations used here, see <http://functions.wolfram.com/Notations/>.

Please cite this document by referring to the functions.wolfram.com page from which it was downloaded, for example:

<http://functions.wolfram.com/Constants/E/>

To refer to a particular formula, cite functions.wolfram.com followed by the citation number.

e.g.: <http://functions.wolfram.com/01.03.03.0001.01>

This document is currently in a preliminary form. If you have comments or suggestions, please email comments@functions.wolfram.com.

© 2001-2008, Wolfram Research, Inc.