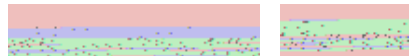


PartitionsP



Notations

Traditional name

Number of unrestricted partitions of an integer

Traditional notation

$p_{\mathbb{H}\mathbb{L}}$

Mathematica StandardForm notation

PartitionsP@nD

Primary definition

$$p_{\mathbb{H}\mathbb{L}} \stackrel{04.16.02.0001.01}{\sim} \left(\text{Dä}^n \frac{1}{\prod_{k=1}^{\infty} (1 - t^k)} \right); n \hat{\in} \mathbb{N}$$

$p_{\mathbb{H}\mathbb{L}}$ is the number of unrestricted partitions of the positive integer n into a sum of strictly positive numbers which add up to n independent of order, when repetitions are allowed.

For example, $p_{\mathbb{H}\mathbb{L}} \stackrel{04.16.02.0002.01}{\sim} 7$. There are 7 possibilities to express 5 as a sum of positive integers:

$5 \stackrel{04.16.02.0002.01}{\sim} 1+4 \stackrel{04.16.02.0002.01}{\sim} 2+3 \stackrel{04.16.02.0002.01}{\sim} 1+1+3 \stackrel{04.16.02.0002.01}{\sim} 1+2+2 \stackrel{04.16.02.0002.01}{\sim} 1+1+1+2 \stackrel{04.16.02.0002.01}{\sim} 1+1+1+1+1$.

$$p_{\mathbb{H}\mathbb{L}} \stackrel{04.16.02.0002.01}{\sim} 0; n \hat{\in} \mathbb{Z} \mathbb{B} n < 0$$

Specific values

Values at fixed points

$$p_{\mathbb{H}\mathbb{L}} \stackrel{04.16.03.0001.01}{\sim} 1$$

$$p_{\mathbb{H}\mathbb{L}} \stackrel{04.16.03.0002.01}{\sim} 1$$

$$p_{\mathbb{H}\mathbb{L}} \stackrel{04.16.03.0003.01}{\sim} 2$$

04.16.03.0004.01
 $p\mathbb{H}L\check{S}$ 3

04.16.03.0005.01
 $p\mathbb{H}L\check{S}$ 5

04.16.03.0006.01
 $p\mathbb{H}L\check{S}$ 7

04.16.03.0007.01
 $p\mathbb{H}L\check{S}$ 11

04.16.03.0008.01
 $p\mathbb{H}L\check{S}$ 15

04.16.03.0009.01
 $p\mathbb{H}L\check{S}$ 22

04.16.03.0010.01
 $p\mathbb{H}L\check{S}$ 30

04.16.03.0011.01
 $p\mathbb{H}OL\check{S}$ 42

04.16.03.0013.01
 $p\mathbb{H}OL\check{S}$ 42

04.16.03.0014.01
 $p\mathbb{H}1L\check{S}$ 56

04.16.03.0015.01
 $p\mathbb{H}2L\check{S}$ 77

04.16.03.0016.01
 $p\mathbb{H}3L\check{S}$ 101

04.16.03.0017.01
 $p\mathbb{H}4L\check{S}$ 135

04.16.03.0018.01
 $p\mathbb{H}5L\check{S}$ 176

04.16.03.0019.01
 $p\mathbb{H}6L\check{S}$ 231

04.16.03.0020.01
 $p\mathbb{H}7L\check{S}$ 297

04.16.03.0021.01
 $p\mathbb{H}8L\check{S}$ 385

04.16.03.0022.01
 $p\mathbb{H}9L\check{S}$ 490

04.16.03.0023.01
 $p\mathbb{H}OL\check{S}$ 627

04.16.03.0024.01
 $p\mathbb{H}1L\check{S}$ 792

04.16.03.0025.01
 $p\mathbb{E}2L\check{S}$ 1002

04.16.03.0026.01
 $p\mathbb{E}3L\check{S}$ 1255

04.16.03.0027.01
 $p\mathbb{E}4L\check{S}$ 1575

04.16.03.0028.01
 $p\mathbb{E}5L\check{S}$ 1958

04.16.03.0029.01
 $p\mathbb{E}6L\check{S}$ 2436

04.16.03.0030.01
 $p\mathbb{E}7L\check{S}$ 3010

04.16.03.0031.01
 $p\mathbb{E}8L\check{S}$ 3718

04.16.03.0032.01
 $p\mathbb{E}9L\check{S}$ 4565

04.16.03.0033.01
 $p\mathbb{B}0L\check{S}$ 5604

04.16.03.0034.01
 $p\mathbb{B}1L\check{S}$ 6842

04.16.03.0035.01
 $p\mathbb{B}2L\check{S}$ 8349

04.16.03.0036.01
 $p\mathbb{B}3L\check{S}$ 10 143

04.16.03.0037.01
 $p\mathbb{B}4L\check{S}$ 12 310

04.16.03.0038.01
 $p\mathbb{B}5L\check{S}$ 14 883

04.16.03.0039.01
 $p\mathbb{B}6L\check{S}$ 17 977

04.16.03.0040.01
 $p\mathbb{B}7L\check{S}$ 21 637

04.16.03.0041.01
 $p\mathbb{B}8L\check{S}$ 26 015

04.16.03.0042.01
 $p\mathbb{B}9L\check{S}$ 31 185

04.16.03.0043.01
 $p\mathbb{H}0L\check{S}$ 37 338

04.16.03.0044.01
 $p\mathbb{H}1L\check{S}$ 44 583

04.16.03.0045.01
 $p_{\mathbb{H}2L\check{S}}$ 53 174

04.16.03.0046.01
 $p_{\mathbb{H}3L\check{S}}$ 63 261

04.16.03.0047.01
 $p_{\mathbb{H}4L\check{S}}$ 75 175

04.16.03.0048.01
 $p_{\mathbb{H}5L\check{S}}$ 89 134

04.16.03.0049.01
 $p_{\mathbb{H}6L\check{S}}$ 105 558

04.16.03.0050.01
 $p_{\mathbb{H}7L\check{S}}$ 124 754

04.16.03.0051.01
 $p_{\mathbb{H}8L\check{S}}$ 147 273

04.16.03.0052.01
 $p_{\mathbb{H}9L\check{S}}$ 173 525

04.16.03.0053.01
 $p_{\mathbb{H}0L\check{S}}$ 204 226

Values at infinities

04.16.03.0012.01
 $p_{\mathbb{H}L\check{S}}$ ∞

General characteristics

Domain and analyticity

The partitions $p_{\mathbb{H}L}$ is a nonanalytical function which is defined only for integers.

04.16.04.0001.01
 $n^{\text{TM}} p_{\mathbb{H}L} > \mathbb{N}^{\text{TM}} \mathbb{N}^+$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

04.16.06.0001.01

$$p_{\text{H}}L\check{S} \frac{1}{p\sqrt{2}} \hat{a} \sum_{k=1}^{\infty} \frac{A_{\text{H}, nL} \sqrt{k} \left(\sinh \left(\frac{1}{k} p \sqrt{\frac{2}{3}} \sqrt{n - \frac{1}{24}} \right) J_{n - \frac{1}{24}} \tilde{N}^{\frac{1}{2}} \right)}{\Gamma(n)}$$

$$A_{\text{H}, nL} \check{S} \hat{a} \sum_{h=1}^k d_{\text{gcdH}, kL, 1} \exp \left(p \hat{a} \sum_{j=1}^{k-1} \frac{1}{k} j \left(\frac{hj}{k} - \left\lfloor \frac{hj}{k} \right\rfloor - \frac{1}{2} \right) - \frac{2p\hat{a}hn}{k} \right)$$

04.16.06.0003.01

$$p_{\text{H}}L\check{S} \frac{p^2}{9\sqrt{3}} \hat{a} \sum_{k=1}^{\infty} \frac{A_{\text{H}, nL}}{k^{5/2}} {}_0F_1 \left(\frac{5}{2}; \frac{J_{n - \frac{1}{24}} \tilde{N} p^2}{6k^2} \right) \cdot A_{\text{H}, nL} \check{S} \hat{a} \sum_{h=1}^k d_{\text{gcdH}, kL, 1} \exp \left(p \hat{a} \sum_{j=1}^{k-1} \frac{1}{k} j \left(\frac{hj}{k} - \left\lfloor \frac{hj}{k} \right\rfloor - \frac{1}{2} \right) - \frac{2p\hat{a}hn}{k} \right)$$

Asymptotic series expansions

04.16.06.0002.01

$$p_{\text{H}}L\mu \frac{1}{4n\sqrt{3}} \exp \left(\sqrt{\frac{2}{3}} \sqrt{n} p \right) \left(1 + O\left(\frac{1}{n}\right) \right) \cdot \text{H} \otimes \mathbb{N}$$

Generating functions

04.16.11.0001.01

$$p_{\text{H}}L\check{S} \left(\sum_{k=1}^{\infty} \frac{1}{1-t^k} \right) \cdot n \hat{I} \mathbb{N}$$

04.16.11.0002.01

$$p_{\text{H}}L\check{S} \left(\sum_{k=-\infty}^{\infty} \frac{1}{\hat{a} H 1L^k t^{\frac{1}{2} I_3 k^2 + kM}} \right) \cdot n \hat{I} \mathbb{N}$$

04.16.11.0003.01

$$p_{\text{H}}L\check{S} \left(\sum_{k=1}^{\infty} \frac{2 \sqrt[8]{t}}{\sqrt{J_1^{\circ} 10, \sqrt{t} N}} \right) \cdot n \hat{I} \mathbb{N}$$

Identities

Functional identities

04.16.17.0001.01

$$p_{\text{H}}L\check{S} \frac{1}{n} \hat{a} \sum_{k=1}^n s_1 \text{H}L p_{\text{H}} - kL$$

04.16.17.0002.01

$$p_{\text{H}}L\check{S} \hat{a} \sum_{k=1}^n H 1L^{k-1} \left(p \left(n - \frac{1}{2} I_3 k^2 - kM \right) + p \left(n - \frac{1}{2} I_3 k^2 + kM \right) \right)$$

04.16.17.0003.01

$$\sum_{k=1}^n \frac{1}{k} \left(\sqrt{4k^2 - k + 2n + 1} - \sqrt{4k^2 + k + 2n + 1} \right) + \sum_{k=1}^n \frac{1}{k} \left(\sqrt{4k^2 + 3k + n} - \sqrt{4k^2 - 3k + n} \right)$$

Complex characteristics

Real part

04.16.19.0001.01

$$\operatorname{Re} z$$

Imaginary part

04.16.19.0002.01

$$\operatorname{Im} z$$

Absolute value

04.16.19.0003.01

$$|z|$$

Argument

04.16.19.0004.01

$$\arg z$$

Conjugate value

04.16.19.0005.01

$$\bar{z}$$

Summation

Finite summation

04.16.23.0001.01

$$\sum_{k=\lfloor -\frac{1}{6}(\sqrt{24n+1}+1) \rfloor}^{\lfloor \frac{1}{6}(\sqrt{24n+1}-1) \rfloor} \left(n - \frac{1}{2}k(k+1) \right)$$

Infinite summation

04.16.23.0002.01

$$\sum_{k=0}^{\infty} \frac{1}{1-t^k}$$

Representations through equivalent functions

With related functions

04.16.27.0001.01

$$p_{\mathbb{H}}(n) \sim \frac{f_2^n}{k=0} q_{\mathbb{H}} - 2kL_{p_{\mathbb{H}}}$$

Inequalities

04.16.29.0001.01

$$p_{\mathbb{H}}(n) \leq \frac{1}{2} p_{\mathbb{H}}(n-1) + p_{\mathbb{H}}(n+1); n \in \mathbb{N}^+$$

Other identities

Congruence properties

04.16.32.0001.01

$$p_{\mathbb{H}}(n+4) \equiv p_{\mathbb{H}}(n) \pmod{5}$$

04.16.32.0002.01

$$p_{\mathbb{H}}(n+5) \equiv p_{\mathbb{H}}(n) \pmod{7}$$

04.16.32.0003.01

$$p_{\mathbb{H}}(n+6) \equiv p_{\mathbb{H}}(n) \pmod{11}$$

04.16.32.0004.01

$$p_{\mathbb{H}}(n) \equiv p_{\mathbb{H}}(n) \pmod{5^{k_1} 7^{k_2+1} 11^{k_3}}; n \in \mathbb{N}; k_1, k_2, k_3 \in \mathbb{N}$$

History

- G. W. Leibniz (1669) investigated the number of ways a given positive integer can be decomposed into smaller ones
- L. Euler (1740)
- S. Ramanujan (1917)
- G.H. Hardy (1920) introduced the notation $p_{\mathbb{H}}(n)$

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