PartitionsP

Notations

Traditional name

Number of unrestricted partitions of an integer

Traditional notation

$p(n)$

Mathematica StandardForm notation

`PartitionsP[n]`

Primary definition

04.16.02.0001.01

$$p(n) = \left([t^n] \prod_{k=1}^{\infty} \frac{1}{1-t^k}\right) /; n \in \mathbb{N}$$

$p(n)$ is the number of unrestricted partitions of the positive integer n into a sum of strictly positive numbers which add up to n independent of order, when repetitions are allowed.

For example, $p(5) = 7$. There are 7 possibilities to express 5 as a sum of positive integers:
$5 = 1+4 = 2+3 = 1+1+3 = 1+2+2 = 1+1+1+2 = 1+1+1+1+1$.

04.16.02.0002.01

$$p(n) = 0 /; n \in \mathbb{Z} \land n < 0$$

Specific values

Values at fixed points

04.16.03.0001.01

$$p(0) = 1$$

04.16.03.0002.01

$$p(1) = 1$$

04.16.03.0003.01

$$p(2) = 2$$

04.16.03.0004.01

$$p(3) = 3$$

04.16.03.0005.01

$$p(4) = 5$$

04.16.03.0006.01

$$p(5) = 7$$

04.16.03.0007.01

$$p(6) = 11$$

04.16.03.0008.01

$$p(7) = 15$$

04.16.03.0009.01

$$p(8) = 22$$

04.16.03.0010.01

$$p(9) = 30$$

04.16.03.0011.01

$$p(10) = 42$$

04.16.03.0013.01

$$p(10) = 42$$

04.16.03.0014.01

$$p(11) = 56$$

04.16.03.0015.01

$$p(12) = 77$$

04.16.03.0016.01

$$p(13) = 101$$

04.16.03.0017.01

$$p(14) = 135$$

04.16.03.0018.01

$$p(15) = 176$$

04.16.03.0019.01

$$p(16) = 231$$

04.16.03.0020.01

$$p(17) = 297$$

04.16.03.0021.01

$$p(18) = 385$$

04.16.03.0022.01

$$p(19) = 490$$

04.16.03.0023.01

$$p(20) = 627$$

04.16.03.0024.01

$$p(21) = 792$$

04.16.03.0025.01
$p(22) = 1002$

04.16.03.0026.01
$p(23) = 1255$

04.16.03.0027.01
$p(24) = 1575$

04.16.03.0028.01
$p(25) = 1958$

04.16.03.0029.01
$p(26) = 2436$

04.16.03.0030.01
$p(27) = 3010$

04.16.03.0031.01
$p(28) = 3718$

04.16.03.0032.01
$p(29) = 4565$

04.16.03.0033.01
$p(30) = 5604$

04.16.03.0034.01
$p(31) = 6842$

04.16.03.0035.01
$p(32) = 8349$

04.16.03.0036.01
$p(33) = 10\,143$

04.16.03.0037.01
$p(34) = 12\,310$

04.16.03.0038.01
$p(35) = 14\,883$

04.16.03.0039.01
$p(36) = 17\,977$

04.16.03.0040.01
$p(37) = 21\,637$

04.16.03.0041.01
$p(38) = 26\,015$

04.16.03.0042.01
$p(39) = 31\,185$

04.16.03.0043.01
$p(40) = 37\,338$

04.16.03.0044.01
$p(41) = 44\,583$

04.16.03.0045.01
$p(42) = 53174$

04.16.03.0046.01
$p(43) = 63261$

04.16.03.0047.01
$p(44) = 75175$

04.16.03.0048.01
$p(45) = 89134$

04.16.03.0049.01
$p(46) = 105558$

04.16.03.0050.01
$p(47) = 124754$

04.16.03.0051.01
$p(48) = 147273$

04.16.03.0052.01
$p(49) = 173525$

04.16.03.0053.01
$p(50) = 204226$

Values at infinities

04.16.03.0012.01
$p(\infty) = \infty$

General characteristics

Domain and analyticity

The partitions $p(n)$ is a nonanalytical function which is defined only for integers.

04.16.04.0001.01
$n \in p(n) \longrightarrow \mathbb{N} \in \mathbb{N}^+$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

04.16.06.0001.01

$$p(n) \propto \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A(k, n) \sqrt{k}\, \frac{\partial \left(\sinh\left(\frac{1}{k} \pi \sqrt{\frac{2}{3}} \sqrt{n - \frac{1}{24}} \right) \left(n - \frac{1}{24}\right)^{-\frac{1}{2}} \right)}{\partial n} \;/;$$

$$A(k, n) \propto \sum_{h=1}^{k} \delta_{\gcd(h,k),1} \exp\left(\pi\, i \sum_{j=1}^{k-1} \frac{1}{k}\, j \left(\frac{h\, j}{k} - \left\lfloor \frac{h\, j}{k} \right\rfloor - \frac{1}{2} \right) - \frac{2\pi\, i\, h\, n}{k} \right)$$

04.16.06.0003.01

$$p(n) \propto \frac{\pi^2}{9\sqrt{3}} \sum_{k=1}^{\infty} \frac{A(k, n)}{k^{5/2}}\, {}_0F_1\left(; \frac{5}{2}; \frac{\left(n - \frac{1}{24}\right) \pi^2}{6\, k^2} \right) /;\; A(k, n) \propto \sum_{h=1}^{k} \delta_{\gcd(h,k),1} \exp\left(\pi\, i \sum_{j=1}^{k-1} \frac{1}{k}\, j \left(\frac{h\, j}{k} - \left\lfloor \frac{h\, j}{k} \right\rfloor - \frac{1}{2} \right) - \frac{2\pi\, i\, h\, n}{k} \right)$$

Asymptotic series expansions

04.16.06.0002.01

$$p(n) \propto \frac{1}{4\, n\sqrt{3}} \exp\left(\sqrt{\frac{2}{3}} \sqrt{n}\, \pi \right) \left(1 + O\left(\frac{1}{n} \right) \right) /;\; (n \to \infty)$$

Generating functions

04.16.11.0001.01

$$p(n) \propto \left([t^n] \prod_{k=1}^{\infty} \frac{1}{1 - t^k} \right) /;\; n \in \mathbb{N}$$

04.16.11.0002.01

$$p(n) \propto \left([t^n]\, 1 / \left(\sum_{k=-\infty}^{\infty} (-1)^k\, t^{\frac{1}{2}(3 k^2 + k)} \right) \right) /;\; n \in \mathbb{N}$$

04.16.11.0003.01

$$p(n) \propto \left([t^n] \sqrt[3]{\frac{2 \sqrt[8]{t}}{\vartheta_1'\left(0, \sqrt{t}\right)}} \right) /;\; n \in \mathbb{N}$$

Identities

Functional identities

04.16.17.0001.01

$$p(n) \propto \frac{1}{n} \sum_{k=1}^{n} \sigma_1(k)\, p(n - k)$$

04.16.17.0002.01

$$p(n) \propto \sum_{k=1}^{n} (-1)^{k-1} \left(p\left(n - \frac{1}{2}(3 k^2 - k) \right) + p\left(n - \frac{1}{2}(3 k^2 + k) \right) \right)$$

04.16.17.0003.01

$$p(2n+1) = p(n) - \sum_{k=1}^{\infty} (-1)^k \left(p\left(-3k^2 - k + 2n + 1\right) + p\left(-3k^2 + k + 2n + 1\right)\right) + \sum_{k=1}^{\infty} \left(p\left(-4k^2 + 3k + n\right) + p\left(-4k^2 - 3k + n\right)\right)$$

Complex characteristics

Real part

04.16.19.0001.01

$$\operatorname{Re}(p(n)) = p(n)$$

Imaginary part

04.16.19.0002.01

$$\operatorname{Im}(p(n)) = 0$$

Absolute value

04.16.19.0003.01

$$|p(n)| = p(n)$$

Argument

04.16.19.0004.01

$$\arg(p(n)) = 0$$

Conjugate value

04.16.19.0005.01

$$\overline{p(n)} = p(n)$$

Summation

Finite summation

04.16.23.0001.01

$$\sum_{k=\left\lceil -\frac{1}{6}\left(\sqrt{24n+1}+1\right)\right\rceil}^{\left\lfloor \frac{1}{6}\left(\sqrt{24n+1}-1\right)\right\rfloor} (-1)^k \, p\left(n - \frac{1}{2} k (3k+1)\right) = 0$$

Infinite summation

04.16.23.0002.01

$$\sum_{k=0}^{\infty} p(k) \, t^k = \prod_{k=1}^{\infty} \frac{1}{1 - t^k}$$

Representations through equivalent functions

With related functions

04.16.27.0001.01

$$p(n) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} q(n-2k)\, p(k)$$

Inequalities

04.16.29.0001.01

$$p(n) \leq \frac{1}{2}\left(p(n-1) + p(n+1)\right) /;\ n \in \mathbb{N}^+$$

Other identities

Congruence properties

04.16.32.0001.01

$$p(5\,n + 4) \bmod 5 = 0$$

04.16.32.0002.01

$$p(7\,n + 5) \bmod 7 = 0$$

04.16.32.0003.01

$$p(11\,n + 6) \bmod 11 = 0$$

04.16.32.0004.01

$$p(n) \bmod \left(5^{k_1}\, 7^{k_2+1}\, 11^{k_3}\right) = 0 /;\ (24\,n) \bmod \left(5^{k_1}\, 7^{k_2}\, 11^{k_3}\right) = 1 \land k_1 \in \mathbb{N} \land k_2 \in \mathbb{N} \land k_3 \in \mathbb{N}$$

History

– G. W. Leibniz (1669) investigated the number of ways a given positive integer can be decomposed into smaller ones

– L. Euler (1740)

– S. Ramanujan (1917)

– G.H. Hardy (1920) introduced the notation $p(n)$

Copyright