Max

Notations

Traditional name

Maximum

Traditional notation

$\max(x_1, x_2, \ldots)$

Mathematica StandardForm notation

`Max[`x_1`, `x_2`, …]`

Primary definition

01.34.02.0001.01

$$\max(x_1, x_2) = \frac{1}{2}\left(x_1 + x_2 + \sqrt{(x_1 - x_2)^2}\right) /; x_1 \in \mathbb{R} \land x_2 \in \mathbb{R}$$

01.34.02.0002.01

$$\max(x_1, x_2, \ldots, x_n) = \max(\max(x_1, x_2), x_3, \ldots, x_n)$$

$\max(x_1, x_2, \ldots)$ is the numerically largest of the real numbers x_k.

$\max(z_1, z_2, \ldots)$ is not defined for complex numbers z_k.

Specific values

Specialized values

01.34.03.0001.01

$$\max(x) = x$$

01.34.03.0002.01

$$\max(x_1, x_1, \ldots, x_1) = x_1$$

01.34.03.0003.01

$$\max(x_1, x_2, x_3) = x_1 /; x_1 \in \mathbb{R} \land x_2 \in \mathbb{R} \land x_3 \in \mathbb{R} \land x_1 \geq x_2 \land x_1 \geq x_3$$

Values at fixed points

01.34.03.0004.01

$$\max() = -\infty$$

01.34.03.0005.01

$$\max(1, 3) = 3$$

01.34.03.0006.01

$$\max(2, 6, 1, \pi, 8, 12, 8, 2, 6, 4, 6, 9) = 12$$

Values at infinities

01.34.03.0007.01

$$\max(\infty, x_2, \ldots, x_n) = \infty$$

01.34.03.0008.01

$$\max(-\infty, x_2, \ldots, x_n) = \max(x_2, \ldots, x_n)$$

01.34.03.0009.01

$$\max(-\infty, \infty) = \infty$$

General characteristics

Domain and analyticity

max is real valued function of an arbitrary number of real variables. In $\mathbb{R}^n$ it is a piecewise linear function. The derivative of $\max(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n)$ is discontinuous at $x_j = x_k$ for all j, k.

01.34.04.0001.01

$$(x_1 * x_2 * \ldots * x_n) \to \max(x_1, x_2, \ldots, x_n) : \mathbb{R}^n \to \mathbb{R}$$

Symmetries and periodicities

Permutation symmetry

01.34.04.0002.01

$$\max(x_1, x_2) = \max(x_2, x_1)$$

01.34.04.0003.01

$$\max(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n) = \max(x_1, x_2, \ldots, x_j, \ldots, x_k, \ldots, x_n)$$

Periodicity

No periodicity

Sets of discontinuity

The function $\max(x_1, x_2, \ldots, x_n)$ is continuous function in $\mathbb{R}^n$.

01.34.04.0004.01

$$\mathcal{DS}_{x_k}(\max(x_1, x_2, \ldots, x_n)) = \{\} \,/;\, 1 \le k \le n$$

Limit representations

01.34.09.0001.01

$$\max(x_1, x_2) = \lim_{\epsilon \to 0} \epsilon \log\left(e^{\frac{x_1}{\epsilon}} + e^{\frac{x_2}{\epsilon}}\right)$$

01.34.09.0002.01

$$\max(x_1, x_2, \ldots, x_n) = \lim_{z \to \infty} \left(\frac{1}{n} \sum_{k=1}^{n} x_k^z \right)^{1/z}$$

Transformations

Transformations and argument simplifications

01.34.16.0001.01

$$\max(-x_1, -x_2, \ldots, -x_n) = -\min(x_1, x_2, \ldots, x_n)$$

01.34.16.0002.01

$$\max(|x|, -|x|) = |x|$$

Identities

Functional identities

01.34.17.0001.01

$$\max(x_1, x_2) = \max(x_2, x_1)$$

01.34.17.0002.01

$$\max(x_1, x_2, x_3, \ldots) = \max(x_1, \max(x_2, x_3, \ldots))$$

Complex characteristics

Real part

01.34.19.0001.01

$$\operatorname{Re}(\max(x_1, x_2, \ldots, x_n)) = \max(x_1, x_2, \ldots, x_n)$$

Imaginary part

01.34.19.0002.01

$$\operatorname{Im}(\max(x_1, x_2, \ldots, x_n)) = 0$$

Absolute value

01.34.19.0003.01

$$|\max(x_1, x_2, \ldots, x_n)| = \sqrt{\max(x_1, x_2, \ldots, x_n)^2}$$

Argument

01.34.19.0004.01

$$\arg(\max(x_1, x_2, \ldots, x_n)) = \tan^{-1}(\max(x_1, x_2, \ldots, x_n), 0)$$

Conjugate value

01.34.19.0005.01

$$\overline{\max(x_1, x_2, \ldots, x_n)} = \max(x_1, x_2, \ldots, x_n)$$

Summation

01.34.23.0001.01

$$\sum_{k=0}^{m-1}\sum_{l=0}^{n-1}\max\left(\frac{k}{m},\frac{l}{n}\right) \unicode{x2A75} \frac{2\,m\,n}{3}-\frac{m+n+1}{4}-\frac{m^2+n^2-\gcd(m,n)^2}{12\,m\,n} \;/;\; m\in\mathbb{N}^+ \land n\in\mathbb{N}^+$$

01.34.23.0002.01

$$\sum_{j=0}^{m-1}\sum_{k=0}^{n-1}\sum_{l=0}^{o-1}\max\left(\frac{j}{m},\frac{k}{n},\frac{l}{o}\right) \unicode{x2A75} \frac{3\,m\,n\,o}{4}-\frac{1}{8}(m+n+o+1)+\frac{2\,o\,m+m+o}{24\,n}-\frac{1}{6}(m\,n+o\,n+m\,o)+\frac{2\,o\,n+n+o}{24\,m}+\frac{2\,n\,m+m+n}{24\,o}-\frac{(n+1)\gcd(m,o)^2}{24\,m\,o}-\frac{(m+1)\gcd(n,o)^2}{24\,n\,o}-\frac{(o+1)\gcd(m,n)^2}{24\,m\,n} \;/;\; m\in\mathbb{N}^+ \land n\in\mathbb{N}^+ \land o\in\mathbb{N}^+$$

Integral transforms

Fourier exp transforms

01.34.22.0001.01

$$\mathcal{F}_{(t_1,t_2)}[\max(t_1,t_2)](z_1,z_2) \unicode{x2A75} -i\,\pi\,(\delta(z_2)\,\delta'(z_1)+\delta(z_1)\,\delta'(z_2))-\frac{\delta(z_1+z_2)}{z_1^2}$$

Laplace transforms

01.34.22.0002.01

$$\mathcal{L}_{(t_1,t_2)}[\max(t_1,t_2)](z_1,z_2) \unicode{x2A75} \frac{z_1^2+z_2\,z_1+z_2^2}{z_1^2\,z_2^2\,(z_1+z_2)}$$

Representations through more general functions

Through other functions

01.34.26.0001.01

$$\max(x_1,x_2) \unicode{x2A75} \frac{1}{2}\left(x_1+x_2+\sqrt{(x_1-x_2)^2}\right)$$

Representations through equivalent functions

01.34.27.0001.01

$$\max(x_1,x_2) \unicode{x2A75} x_2+(x_1-x_2)\,\theta(x_1-x_2)$$

01.34.27.0002.01

$$\max(x_1,x_2,x_3) \unicode{x2A75} x_2+(x_1-x_2)\,\theta(x_1-x_2)+\theta(x_3-x_2)\,(\theta(x_2-x_1)+\theta(x_3-x_1)\,\theta(x_1-x_2))\,(x_3-x_2-(x_1-x_2)\,\theta(x_1-x_2))$$

01.34.27.0003.01

$$\max(x_1, x_2, \ldots, x_n) \equiv \sum_{k_1=1}^{n} x_{k_1} - \sum_{k_1=1}^{n} \sum_{k_2=k_1+1}^{n} \min(x_{k_1}, x_{k_2}) + \sum_{k_1=1}^{n} \sum_{k_2=k_1+1}^{n} \sum_{k_3=k_2+1}^{n} \min(x_{k_1}, x_{k_2}, x_{k_3}) - \ldots +$$

$$(-1)^{j+1} \sum_{k_1=1}^{n} \sum_{k_2=k_1+1}^{n} \ldots \sum_{k_j=k_{j-1}+1}^{n} \min(x_{k_1}, x_{k_2}, \ldots, x_{k_j}) + \ldots + (-1)^{n+1} \sum_{k_1=1}^{n} \sum_{k_2=k_1+1}^{n} \ldots \sum_{k_n=k_{n-1}+1}^{n} \min(x_{k_1}, x_{k_2}, \ldots, x_{k_n})$$

History

The function max is encountered often in mathematics and the natural sciences.

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